

## Relationship between active learning heuristic problem-solving approach and students' attitude towards mathematics

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### Abstract

The study explored the direct relationship between active learning heuristic problem-solving approach and students' attitude towards mathematics, using linear programming (LP) word tasks. Two instruments were used for data collection: the Attitude towards Mathematics Inventory-Short Form was adapted (with  $\alpha = .75$ ) as a multidimensional measurement tool, and a validated standardized active learning heuristic problem-solving tool. A quantitative approach with a quasi-experimental pre-test, post-test, and non-equivalent control group study design was adopted. A sample of 608 grade 11 Ugandan students (291 male and 317 female) from eight secondary schools (public and private), four from central Uganda, and the remaining four from eastern Uganda participated. Data were analyzed using PROCESS macro for SPSS (v.4). The results revealed a direct significant positive relationship between active learning heuristic problem solving approach (ALHPSA) and students' attitude towards solving LP word tasks (ATLPWTs). Thus, the ALHPSA positively and directly impacted on students' ATLPWTs. The findings can be explained by theoretical, conceptual, and/or psychometric factors. Overall, the results indicate beneficial practical implications that support the theoretical framework for enhancing the learning of mathematics using word problems in Ugandan secondary schools and beyond.

**Keywords:** active learning, attitude, heuristic problem-solving, mathematics, linear programming, secondary schools

## INTRODUCTION

### Concept of Active Learning Heuristic Problem-Solving

The term active learning in mathematics education encompasses different instructional strategies that promote students' engagement and active participation in constructing knowledge and understanding of particular mathematics concepts. These strategies may take the form of hands-on activities, problem-solving tasks, critical thinking, etc. The active learning approach involves learners' individual or collaborative performance on routine and non-routine tasks. In this perspective, learners are directly engaged in thinking and solving mathematics through discussions, reviews, evaluations, concept maps, role plays, hands-on projects, and cooperative group studies (Prince, 2004). Often,

active learning tasks require learners to make their thinking explicit, allowing educators to gauge and understand students' learning. This approach generally engages students in deep and broader learning and builds learners' higher-order thinking skills. This may consequently develop a positive attitude towards learning as opposed to learners being passive listeners. The main purpose is to develop students' higher-order thinking skills (analysis, synthesis, and evaluation) and prepare them to apply mathematics in real-life scenarios. Studies have shown that students in typical active learning classrooms perform better than those taught conventionally (Freeman et al., 2014; Prince, 2004). This is because students have an opportunity to reflect, conjecture, or predict outcomes, and then to share and discuss the concepts learnt with teachers and their peers to activate and re-activate their cognitive processes. Active learning also helps students to reflect on their

### Contribution to the literature

- The study explored the relationship between active learning heuristic problem-solving approach and students' attitude towards mathematics using LP word tasks.
- This study provides insight in learning mathematics in the sense that the more actively learners are engaged in learning mathematics, the better their attitude towards mathematics.
- The results indicate practical implications that support the theoretical framework (constructivism) for enhancing the learning of mathematics through a problem-based approach using LP word problems.
- Problem-solving is one of the 21st century skills, and is practiced in finding optimal solutions to non-routine mathematics problems which directly relate to real-life situations.

understanding by encouraging them to make connections between prior mathematical knowledge and understanding, and linking it to the learning of new concepts.

Research shows that non-routine mathematical problems are cognitively more challenging and demanding than routine problems (Chong et al., 2018; Lester, 2013; Mogari & Chirove, 2017; NCTM, 2014). Unlike routine mathematics tasks that are usually conceptual and are answered by applying clearly defined and generally acceptable mathematical rules and principles, non-routine mathematics tasks involve the use of cognitive and meta-cognitive strategies (Mogari & Chirove, 2017). These strategies do not necessarily guarantee the solution to the tasks (problems) but may help to establish effective procedures for finding approximate solutions (Abel, 2003). During problem solving, one may choose the path that seems to result in some progress towards the goal. Such a rule is an example of a heuristic. According to Abel (2003), the term heuristic refers to a rule of thumb that serve as a guide to PS processes.

In human decision-making, heuristics are integrated into PS and are referred to as exploratory PS techniques (Gurat, 2018). Heuristics are approaches to PS and encompasses students' active involvement and self-discovery through experience as guided by educators. For instance, finding an optimal solution (e.g., to LP tasks) may be hard or impractical. The active learning heuristic PS strategies (**Appendix A**) may be handy in finding satisfactory solutions. According to Polya (2004), PS heuristics involve understanding the problem, devising the plan, carrying out the plan, and looking back. To adequately solve any mathematics problem, learners should first understand the verbal statement of the problem (what is known or unknown?) and develop the desire to find the solution as a prerequisite step and a foundation to PS. This step is inevitable in the sense that failure to understand the problem yields wrong optimal solutions. However, some students experience cognitive challenges of converting textual information into conceptual understanding. Teachers should adequately address this challenge (gap) before solving non-routine word tasks (e.g., LP) to minimize misconceptions and errors (King, 1991). Heuristics are,

therefore, basic decision-making processes to PS. Once the problem has been understood, it is transformed into schemas by relating to other auxiliary problems solved previously. This may be followed by writing correct procedures, relating the known to other unfamiliar tasks, checking, verifying and proving that the steps are correct, and examining the final solution by checking the arguments, and investigating whether or not the method can be applied to solve related tasks.

Learners who adapt the heuristic PS strategies in learning mathematics are likely to develop PS and critical thinking skills. Research has shown that students who are actively involved in PS achieve more, work collaboratively and retain more (Klang et al., 2021). The student's involvement in active learning process integrated with group work increases conceptual knowledge and understanding, interest, and a positive attitude towards what is learned. Consequently, students' critical thinking skills are enhanced (Hoon et al., 2013). Other benefits of the heuristic PS approach include students' active participation in form of exploratory learning, arousing positive attitude towards specific content (and mathematics generally), development of students' communication and social skills, fostering teamwork, and consequently addressing individual students' learning gaps (Gurat, 2018; Kigamba et al., 2021). Albay (2019) further noted that students' active involvement develops their critical thinking and PS skills. In turn, students work collaboratively, develop teamwork spirit especially when effective PS strategies are applied. In this research, ALHPSA has been operationalized to refer to the application of graphs in finding solutions to word problems as well as using Newman Error Analysis prompts for effective learning of mathematics and LP in particular.

In addition, several educational competencies have been developed in line with PS and the stated 21<sup>st</sup> century learning skills. To achieve these competencies, Griffin et al. (2012) formulated the framework for 21<sup>st</sup> century skills in terms of ways of thinking, ways of working, tools for working, and living in the world. According to Jäder et al. (2020), PS is one of the essential competencies students need to master to fit into today's changing society. According to Martenz (2015), "PS is

the process of moving toward a goal when the path to that goal is uncertain". Thus, PS may range from solving simple to complex, concrete to abstract problems. However, what a person considers a problem may not necessarily be the problem for the other person. Consequently, the concept of problem-solving is relative and should resonate with the learning needs.

Problem-solving may involve solving both routine and non-routine tasks. This research focused mainly on solution to LP word tasks, which are a subset of non-routine tasks. According to Mogari and Chirove (2017), "non-routine mathematical problems are those that are more complicated and difficult, do not have straightforward solutions, require productive thinking, are approached in more or less sophisticated ways, are nonstandard, involve unexpected and unfamiliar solutions, require an insightful approach and strategic thinking and involve the use of various mathematical concepts" (p. 4527). Therefore, non-routine mathematics tasks are those that are unfamiliar to students, pose cognitive demands, and relate to the application of mathematics in daily contexts and settings. Such tasks require learners' creativity, logical reasoning, higher-order thinking involving application of algebraic concepts, formulae and algorithms (Chong et al., 2018). Thus, learners should have the ability to synthesize and coordinate their knowledge, skills, and understanding and suitably apply them to solving novel problem situations. Additionally, these tasks prepare and foster students' communication skills and confidence for real-life PS scenarios. An example of a non-routine problem is shown in **Appendix B**. This question is a typical LP word task that require students' application of prior conceptual knowledge and understanding of equations and inequalities.

### Students' Attitude Towards Mathematics

The term attitude is the most indispensable concept in contemporary social psychology and science. It is related to emotional and mental entities that drive an individual towards performing a particular task (Perloff, 2016). According to Aiken (1970), attitude is "a learned disposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept or another person" (p. 551). Attitude towards mathematics is viewed as positive, negative, or neutral feelings and dispositions. It can be bi-dimensional (a person's emotions and beliefs) or multidimensional (affect, behavior, and cognition) (Lin & Huang, 2014). Over the last four decades, an extensive body of research from different settings and contexts has investigated variables that influence students' attitude towards science, technology, engineering and mathematics (e.g., Aiken 1970; Arslan et al., 2014; Chen et al., 2018, Davadas & Lay, 2020; Gardner, 1975; Kasimu & Imoro, 2017; Kempa & McGough, 1977; Maamin et al., 2022; Pepin, 2011; Utsumi & Mendes, 2000). This means

that attitude may determine and may be used as a predictor for measuring students' academic achievement. In this study, we were particularly concerned with an exploration of the effect of ALHPSA on students' attitude towards mathematics using LP word tasks. This is due to the significant roles LP plays in constructing elementary and advanced models for understanding science, technology and engineering.

Numerous studies on students' attitude towards mathematics which is always translated as liking or disliking of the subject have been widely investigated and published (e.g., Arslan et al., 2014; Davadas & Lay, 2020; Pepin, 2011; Utsumi & Mendes, 2000). To some secondary school students, mathematics appears abstract, difficult to comprehend, boring is viewed with limited relevance to everyday life experiences. Students start learning the subject well but gradually start disliking some topics or the entire subject. They feel uncomfortable and nervous during learning and examinations. This is partly attributed to students' lack of self-confidence, critical thinking and motivation during PS. To some students, persevering and studying advanced mathematics after the compulsory level has become a nightmare. Indeed, some students do not seem to know the significance of learning mathematics beyond the compulsory level. Students may (may not) relate mathematical concepts beyond the classroom environment if they have a negative attitude towards mathematics. This may lead to students' failure to positively transfer mathematical knowledge and skills in solving real-life problems.

Mathematicians have attempted to research and understand affective variables that significantly influence students' attitude towards mathematics (e.g., Barmby et al., 2008; Davadas & Lay, 2020; Di Martino & Zan, 2011; Evans & Field, 2020; Grootenboer & Hemmings, 2007; Hannula, 2002; Maamin et al., 2022; Marchis, 2011; Pongsakdi et al., 2019; Yasar, 2016; Zan et al., 2006). Some researchers have gone ahead to ask fundamental questions on whether or not students' attitude towards mathematics is a general phenomenon or dependent on some specific variables. To this effect, some empirical findings report students' attitudes towards specific units or topics in mathematics with the main objective of enhancing the learning of particular subject content and mathematics generally (e.g., Arslan et al., 2014; Estrada & Batanero, 2019; Gagatsis & Kyriakides, 2000; Julius et al., 2018; Mumcu & Aktas, 2015; Selkirk, 1975; Townsend & Wilton, 2003).

Rather than investigating students' general attitudes toward mathematics, recent research has also attempted to identify background factors as a basis for understanding students' attitude towards mathematics. Thus, students at different academic levels may have negative or positive attitude towards mathematics due to fundamentally different reasons. Yet, some empirical studies (e.g., Berger et al., 2020; Chen et al., 2018;

Davadas & Lay, 2020; Grootenboer & Hemmings, 2007; Hwang & Son, 2021; Lipnevich et al., 2011; Ma, 1997; Maamin et al., 2022; Mazana et al., 2018; Mulhern & Rae, 1998; Opolot-Okurut, 2010; Sandman, 1980; Tapia, 1996) have shown the existence of a positive relationship between attitude and achievement in mathematics. From the above studies, it appears that multiple factors ranging from students' demographic to teachers' classroom instructional practices influence students' attitude towards learning mathematics.

This paper presents results from a more specific investigation into the effect of ALHPSA on students' attitude towards learning mathematics using LP word tasks (**Appendix B**). This is because studies concerning attitudes towards mathematics have begun to drift from examining general attitudes to a more differentiated conceptualization of specific students' attitude formations, and in different units (topics). Although different standardized attitudinal scales (e.g., Code et al., 2016; Fennema & Sherman, 1976; Tapia, 1996) have been developed, adapted or adopted to measure different variables influencing students' attitude towards mathematics, this study specifically investigated the influence of some of these constructs on students' attitude towards LP.

### **Students' Attitude Towards Mathematics Word Problems**

Verschaffel et al. (2010) define word problems as "verbal descriptions of problem situations where in one or more questions have answers which can be obtained by the application of mathematical operations to numerical data available in the problem statement." The authors view many word problems emanating from real-life world scenarios. Some empirical findings (e.g., Boonen et al., 2016) show that mathematics word problems link school mathematics to real-life world applications. Thus, mathematics word problems play significant roles in equipping learners with the basic knowledge, skills, and, understanding of problem-solving and mathematical modeling. However, the learning of mathematics word problems and related algebraic concepts is greatly affected by students' cognitive and affective factors (Awofala, 2014; Jupri & Drijvers, 2016; Pongsakdi et al., 2020). Research has also shown that mathematics word problems is an area where the majority of students experience learning gaps (Abdullah et al., 2014; Awofala, 2014; Dooren et al., 2018; Goulet-Lyle et al., 2020; Julius et al., 2018a; Pearce et al., 2011; Sa'ad et al., 2014; Verschaffel et al., 2010, 2020a, 2020b). This has generally undermined students' competence, confidence and achievement in mathematics word problems.

Mathematics word problems are intended to help learners to apply mathematics beyond the classroom environment in solving real-life-world problems.

Boonen et al. (2016) and Verschaffel et al. (2020) have argued that mathematics word problems are difficult, complex, and pose comprehension challenges to most learners. This is because word problems require learners to understand and adequately apply previously learned basic algebraic concepts, principles, rules and/or techniques. Indeed, most learners find it difficult to understand the text in the word problems before transformation into models. This is partly due to variation in students' comprehension abilities and language (Strohmaier et al., 2020). Thus, learners may fail to write suitable algebraic symbolic operations and models. Yet, incorrect models lead to wrong algebraic manipulations and consequently wrong graphical representations and optimal solutions.

Notably, research findings by Evans and Field (2020) and Meara et al. (2019) indicate that students' mathematical inefficiency is due to their transitional epistemological and ontological challenges from primary to secondary education. Other studies (e.g., Georgiou et al., 2007; Grootenboer & Hemmings, 2007; Li et al., 2018; Norton, 1998; Sherman, 1979, 1980) have also attributed students' poor performance and achievement in mathematics to gender differences. Thus, students may start learning mathematics well (with positive attitude towards mathematics) from primary but gradually lose interest and confidence in some specific content and finally in mathematics generally. Several strategies should, therefore, be adopted or adapted to boost students' attitude towards specific topics in mathematics. For the case of learning LP word tasks, it is likely that students' attitude towards mathematics (equations and inequalities in particular) gradually drop in favor of other presumably simpler topics. To boost performance in mathematics word problems, Goulet-Lyle et al. (2020) proposed a step-by-step problem-solving strategy to enhance mastery and development of a positive attitude towards learning mathematics.

Students' attitudes should, therefore, be investigated as well as their influence on their conceptual changes. Several empirical studies have also revealed a significant positive relationship between attitude towards, and achievement in mathematics across all levels, and in different contexts (e.g., Bayaga & Wadesango, 2014; Camacho et al., 1998; Chun, 2011; Davadas & Lay, 2020; Karjanto, 2017; Khavenson et al., 2012; Ozdemir & Ovez, 2012; Quaye, 2015; Selkirk, 1975; Tahar et al., 2010; Utsumi & Mendes, 2000; Yáñez-Marquina & Villardón-Gallego, 2016). Many of these studies focused on students' attitude towards mathematics in general, and from the western context. Yet, students may have differing perceptions and attitude towards specific mathematics content irrespective of their setting or context.

## Linear Programming

Linear programming is a classical unit (topic), “a hub” of mathematics word problems - which has gained significant applications in the last decades in mathematics, science, and technology (Aboelmagd, 2018; Colussi et al., 2013; Parlesak et al., 2016; Romeijn et al., 2006). This is because LP links theoretical to practical mathematical applications. The learning of LP provides elementary modeling skills for later applications (Vanderbei, 2014). Previous empirical findings have shown that LP and/or related concepts are not only difficult for learners but also challenging to teach (Awofala, 2014; Goulet-Lyle et al., 2020; Kenney et al., 2020; Verschaffel et al., 2020a, 2020b). Different factors account for learners’ challenges in learning mathematics using word problems (Ahmad et al., 2010; Haghverdi et al., 2012; Heydari et al., 2015). As earlier stated, students’ challenges in learning LP mainly stem from poor comprehension of word problem statements, poor interpretation and inadequate knowledge and understanding of basic algebraic concepts.

To enhance mathematical proficiency, educators should boost students’ cognitive and affective domains in specific mathematics content. This is because the students’ affective domain may directly influence their cognitive and psychomotor domains. In this study, it was predicted that students’ proficiency in solving LP word tasks is largely influenced by their attitude and prior algebraic knowledge, skills, and experiences. Julius et al. (2018a), and (2018b) support this claim and further note that prior conceptual knowledge and understanding coupled with students’ attitude towards mathematics impacts on students’ proficiency in writing relational symbolic mathematical models from word problems, and finding optimal solutions. Despite numerous difficulties encountered by students in learning algebraic inequalities (e.g., Fernández & Molina, 2017; Molina et al., 2017; Bazzini & Tsamir, 2004; Tsamir & Almog, 2001; Tsamir & Bazzini, 2004, 2006; Tsamir & Tirosh, 2006), a combination of methods (strategies) rather than one specific method can be applied to overcome specific students’ learning challenges and/or gaps.

## THEORETICAL FRAMEWORK

This study was situated on the theoretical framework according to constructivism and expectancy-value model of achievement motivation (Wigfield, 1994; Wigfield & Eccles, 2000). The expectancy-value model is based on the expectancy-value theories of achievement. Thus, the theory is based on the premise that success on specific tasks and the values inherent in those tasks is positively correlated with achievement, and consequently students’ abilities. In the context of the attitude towards mathematics inventory-short form (ATMI-SF), the theory combines motivation, enjoyment,

confidence, and usefulness, and related latent variables to explain students’ success in learning mathematics using word tasks. Constructivism is a form of discovery learning which is based on the premise that teachers facilitate learning by actively involving learners to construct their world knowledge and understanding based on individual prior experiences and schema (Olusegun, 2015; Ultanir, 2012). This means that previous knowledge, understanding and reflection with new knowledge is inevitable for subsequent learning and acquisition of both conceptual and procedural knowledge and skills.

This study relates to students’ latent efforts, and persistence, their perceived difficulties and experiences learners encounter when solving LP, a subset of mathematics word tasks. Students’ challenges in learning mathematics using LP word tasks largely depend on their insufficient previous algebraic knowledge and experiences in applying concepts of equations and inequalities to effectively solve real-life world problems. In this article we mainly discuss students’ ATLPTs using the expectancy-value model theory within the constructivism paradigm. Using this paradigm helped to explain the ATMI-SF constructs, and their significance in enhancing the learning of mathematics in secondary schools and beyond. The expectancy-value theory and constructivism have been widely applied to enhance the learning of mathematics and science (Awofala, 2014; Fielding-Wells et al., 2017; Meyer et al., 2019; Wigfield & Eccles, 2000; Yurt, 2015). To foster a positive attitude towards mathematics, teachers should design different tasks for students based on their academic level and previous academic performance. This strategy helps students to effectively apply the previously acquired knowledge, understanding and experiences in subsequent learning. Stein et al. (2000) resonated that students’ proficiency and competency is determined by mathematical tasks they are given. Tasks at the lower cognitive stage (memorization level), must be different from those at the highest cognitive level (doing mathematics). In the context of learning mathematics using LP word tasks, students’ proficiency in finding solutions to equations and inequalities may guarantee mastery of LPMWTs.

## Ugandan Context

In Uganda, studies on predictors of students’ attitude towards science and mathematics are scanty (e.g., Opolot-Okurut, 2010). There are no recent empirical findings on secondary school students’ attitude towards mathematics and mathematics word problems in particular. Solving LP tasks (by graphical method) is one of the topics taught to the 11<sup>th</sup> grade Ugandan lower secondary school students (NCDC, 2008, 2018). Despite students’ general and specific learning challenges in learning mathematics, the objectives of learning LP are embedded within the aims of the Ugandan lower

**Table 1.** Design of the study

| Group        |              | Pre-attitude          | Treatment | Post-attitude          |
|--------------|--------------|-----------------------|-----------|------------------------|
| Experimental | Intact class | Pre-attitude in ALHPS | XTAL      | Post-attitude in ALHPS |
| Control      | Intact class | Pre-attitude in ALHPS | XCAL      | Post-attitude in ALHPS |

Note. XTAL-treatment group: ALHPS & XTAL-control group: No ALHPS

secondary school mathematics curriculum. Some of the specific aims include ... enabling individuals to apply acquired skills and knowledge in solving community problems, instilling a positive attitude towards productive work ... (NCDC, 2018). Generally, the learning of mathematics through LP word problems aims to develop students' critical thinking, PS abilities, application of prior algebraic conceptual knowledge and understanding of linear equations and inequalities in writing models from word problems, and from real-life-world problems. Despite the learning challenges, LP equips learners with adequate knowledge and skills needed for doing advanced mathematics beyond the compulsory level at Uganda Certificate of Education (UCE).

Every academic year, the Uganda National Examinations Board (UNEB) highlights students' strengths and weaknesses in previous examinations at UCE. The consistent reports (UNEB, 2016, 2018, 2019, 2020) on previous examinations on work of candidates show that students' performance in mathematics is not satisfactory especially at distinction level. In particular, previous UNEB examiners' reports show students' poor performance in mathematics word problems. The examination reports further revealed numerous students' specific deficiencies in the topic of LP (**Appendix B**). The above reports and our own experiences as mathematics educators show that students' challenges in LP mainly stem from their inability to comprehend LP word problems, their failure to write correct linear equations and inequalities (in two dimensions) from the given word problem in real-life situations. Thus, wrong models derived from textual questions result in incorrect graphical representations, and consequently wrong optimal solutions. These challenges have consequently hindered students' construction of relevant models in science, mathematics and technology. In addition, learners have consistently demonstrated cognitive obstacles in answering questions on LP, while others elude these questions (**Appendix B**) during national examinations. Noticeably absent in all the UNEB reports are factors that account for students' strengths and weaknesses, and the specific interventions to overcome students' challenges in learning LPMWTs. Some students have even developed a negative attitude towards LP. Yet, students' attitudes may directly impact on their learning outcomes (Code et al., 2016).

Several attitudinal scales (with both cognitive and behavioral components) have been developed (Lim & Chapman, 2013; Yáñez-Marquina & Villardón-Gallego,

2016) adopted or adapted (Lin & Huang, 2014) to assess students' attitude towards mathematics and in specific mathematics content. For instance, geometry attitude scales (Avcu & Avcu, 2015), statistics attitude scales (Ayebo et al., 2019; Khavenson et al., 2012), attitudes toward mathematics word problem inventory (Awofala, 2014), attitude towards geometry inventory instrument (Utlely, 2007), and others. In this study, ATMI-SF instrument (Lin & Huang, 2014) was adapted to investigate the effect of the heuristic problem-solving approach on students' attitude towards mathematics using LP word tasks (**Appendix B**). Taken together, research shows that a high percentage of educational stakeholders around the world are concerned about attitude towards mathematics and word tasks in particular. However, to fully understand students' attitude towards mathematics, it is necessary to investigate beyond general mathematics attitudes and examine specific underlying aspects for these attitudes in the context of learning LP.

## METHODOLOGY

A quantitative survey research design was used to collect, analyze, and describe students' experiences and latent behavior in learning mathematics using LP word tasks.

### Research Design

This study investigated the effect of active learning heuristic problem-solving approach on students' attitude towards mathematics using LP word tasks. The study adopted a quantitative approach to gain a deeper and broader understanding (Creswell, 2014; Creswell, & Plano Clark, 2018; Djamba & Neuman, 2002). A quasi-experimental pre-test, post-test, non-equivalent control group study design was adopted. The stated approach and design were ably used to compare and contrast students' attitude towards learning LP between comparison and experimental groups. Learners from the sampled schools (experimental and comparison groups), and in their intact classes participated. The intact classes were maintained to avoid interfering with the already set respective schools' timetables. **Table 1** depicts the study design.

### Sample and Participants

The sampling frame was the 11<sup>th</sup> grade students for the academic year 2020/2021. Both private and public secondary schools (rural and urban) participated. This study comprised of a research study of 608 11<sup>th</sup> grade

students from eight randomly selected secondary schools. Four schools were selected from Mbale District, eastern Uganda, and four from Mukono District, central Uganda. Schools were allocated to the experimental and comparison groups by a toss of a coin. Two schools from each region were assigned to the experimental group. The selection of the 11 grade students was based on the content covered in this class as outlined by the Ugandan mathematics curriculum materials (NCDC, 2018). Of the 608 grade 11 students, 291 (47.86%) were males and 492 (57.8%) were females with a mean age of 18.36 (standard deviation [SD]=0.94) years. Three hundred seventeen (52.1%) students were from the comparison group while two hundred ninety-one (47.9%) were from the treatment group. The selection of students from the two distant schools within/outside the regions and assigning them to treatment group was to avoid spurious results. The mathematics syllabus containing LP word problems (**Appendix B**) had been completed at the time of data collection. The students were selected to provide their experiences and attitude towards LP word problems. The purpose of the study was clearly explained to the participants. Finally, identification numbers were allotted to participants before they consented. They anonymously and voluntarily completed an adapted ATMI-SF and ALHPSA questionnaires.

### Research Instruments & Procedure for Administration

In addition to demographic questions, ATMI-SF (Lin & Huang, 2014), a 14-item instrument questionnaire consisting of four subscales (enjoyment, motivation, value/usefulness, and self-confidence) was adapted (**Appendix C**) to measure the relationship between ALHPSA and students' attitude towards learning mathematics using LP word tasks. The ATMI-SF (Lim & Chapman, 2013), a 5-point Likert-type scale with response options ranging from "strongly disagree (1)" to "strongly agree (5)" (**Appendix C**) was adapted. The ATMI-SF items were developed and from several mathematics attitudinal questionnaire items (Fennema & Sherman, 1976; Kasimu & Imoro, 2017; Mulhern & Rae, 1998; Primi et al., 2020; Tapia, 1996).

Content validity of the questionnaire was assessed by three experts (one senior teacher for mathematics, one senior lecturer for mathematics education, and one tutor at teacher training institution). The experts were selected based on their vast experience in teaching mathematics at various academic levels. They evaluated the appropriateness and relevance of the adapted questionnaire items. Based on their recommendations, suggestions and comments, some questionnaire items were adjusted to suit students' academic level and language to adequately answer the research objectives. Reliability of the construct was ascertained using a sample of students (n=80) from secondary schools outside the study sample. The internal consistence of the construct was measured using Cronbach's alpha (0.75).

This threshold was acceptable based on recommendations from Hair et al. (2010).

ATMI-SF was then administered to the treatment and comparison groups. Both the experimental and comparison groups were taught mathematics using LP word tasks (equations, inequalities, and LP) following a well designed and approved ugandan mathematics curriculum materials (NCDC, 2018). The experimental group was taught LP using the heuristic PS approach before and after an intervention. The learning in the comparison group was purely conventional, and teachers did not follow proper and organized strategies as it was the case with the treatment group. In particular, students from the treatment group were taught LP using several active learning heuristic strategies following clearly outlined principles and strategies.

### Administration of questionnaires

First, students' prior conceptual knowledge of equations and inequalities plus the basic algebraic principles and understanding were reviewed linking students' previous concepts to the learning of mathematics using LP word tasks. Second, several learning materials were applied to help students adequately master the concepts. The materials included use of graphs, grid boards, excel spreadsheet and GeoGebra software (see **Appendix A** for G01, G02, and G03). These strategies were further integrated in problem solving (Polya, 2014) by ensuring that students understand the LP word problem, devise a plan, effectively carry out the plan and finally look back to verify solution sketches (see **Appendix A** for PS01, PS02, PS03, PS04). To ensure that students minimize occurrence and reoccurrence of errors and misconceptions, the learning of LP was further integrated with Newman Error Analysis (NEA) model (Mushlihah, 2018). The instrument was designed by the researcher and validated by experts in mathematics education as an error analysis tool. It was used by teachers as a framework for explaining the underlying reasons that account for students' incorrect solutions to mathematics using LP word tasks. The teachers mainly emphasized question reading and decoding, comprehension, transformation, process skills and encoding (see **Appendix 3** for NEA01, NEA02, NEA03, NEA04 and NEA05). The participants received the questionnaire items and completed within 30 minutes. All respondents consented to guarantee anonymity and confidentiality.

### Data Analysis

#### Procedure

The ATMI-SF questionnaires were completed by the sampled students in 30 minutes on average at their respective schools. The survey instrument contained a

**Table 2.** Relationship between ALHPS approach & students' ATLPWTs

| Model | R                 | R <sup>2</sup> | Adjusted R <sup>2</sup> | Standard error of the estimate | F change | Sig. F change |
|-------|-------------------|----------------|-------------------------|--------------------------------|----------|---------------|
| 1     | .313 <sup>a</sup> | .098           | .096                    | .45100                         | 65.795   | .000          |

Note. <sup>a</sup>Predictors: (Constant), active learning heuristic problem-solving (ALHPS) & <sup>b</sup>Dependent variable: Attitude towards mathematics word tasks (ATMWTs)

'filter statement', as a social desirability response (SDR) to verify and discard respondents' questionnaires especially those who did not read (see item 15 in **Appendix B**) or finish answering all the questionnaire items (Bäckström & Björklund, 2013; Latkin et al., 2017). Written consent was received from all participants and participation in this study was completely voluntary and confidential. The participants who felt uncomfortable to complete the questionnaires were not penalized. Data were collected with the help of mathematics heads of department who were selected as research assistants on the basis of their expertise and experience. The purpose of the study was explained to the participants before they completed the questionnaire items. The participants completed and returned all the questionnaires to the research assistants. Descriptive and inferential statistics were used to analyze the data. Data were analyzed using the Statistical Package for Social Sciences (SPSS) v.26, with Hayes (2022) PROCESS macro (v.4).

### Ethical considerations

Ethical clearance was sought from the Research and Ethics Committee of the corresponding authors' university. Subsequent permission was sought and granted by the ministry of education and sports, the district education officers, and finally from the headteachers of sampled secondary schools. Upon accessing research participants, the questionnaires were administered to the respondents during school working hours without interfering with the school set timetables. The heads of the sampled schools provided appropriate schedules and personnel to help the principal researcher together with the research assistants to effectively administer the questionnaires. All participants were informed and clearly explained to the purpose of the study before participating. They were assured of confidentiality and, anonymity before they willingly consented and participated. The participants who refused to participate even after the distribution of questionnaires were allowed to withdraw.

## FINDINGS AND INTERPRETATION

### Psychometric Properties

IBM SPSS (v. 26) software package was used for analysis. Preliminary statistical analysis revealed no evidence of missing data due to a few cases which were ignored because they did not exceed 5% of sample cases (Kline Rex, 1998; Lim & Chapman, 2013; Tabachnick & Fidell, 2001). However, out of 639 questionnaires

distributed, 31 questionnaires were removed because the participants did not either conform to SDR (Bäckström & Björklund, 2013; Latkin et al., 2017) or the questionnaires were incomplete. Univariate analysis was run to examine the degree of normality (Hair et al., 2010; Pallant, 2011). The indices for skewness and kurtosis were within the acceptable ranges ( $\pm 2$  and  $\pm 7$ , respectively) (Byrne, 2010; Curran et al., 1996; Hair et al., 2010). Thus, data were fairly normally distributed.

The psychometric properties (reliability and factor analysis) of the two instruments were tested. The Cronbach's alpha coefficient of the adapted ATMI-SF was  $\alpha=0.75$ . Factor analysis was performed using the principal component (with varimax rotation) (Tabachnick & Fidell, 2001; Pallant, 2011; Pituch & Stevens, 2016). The values obtained were consistent with Awofala's (2014) and Lin and Huang's (2014) findings. The Kaiser-Meyer-Olkin measure of sampling adequacy test (KMO) and Bartlett's test of sphericity was conducted. The value of KMO=0.77>0.60; and that of Bartlett's test of sphericity was significant (894.349,  $p<0.05$ ) indicating a substantial correlation in the data and an acceptable fit. For a self-developed standardized ALHPS tool,  $\alpha=0.71$ , KMO=0.74>0.60; and that of Bartlett's test of sphericity was significant (253.092,  $p<0.05$ ). Following the above recommendations, all items were found acceptable with adequate construct validity, internal consistency and homogeneity (Nunnally & Berstain, 1994, Pallant, 2011).

The descriptive statistics (mean, SD, skewness, and kurtosis) were run to show students' scores on ATMI-SF and ATLPWTs. The results did not, however, show any significant differences between ALHPS approach and students' ATLPWTs (enjoyment, motivation, usefulness, and self-confidence). Indeed, both experimental and comparison groups were assumed similar during the pre-test. There was, however, a noticeable change in students' attitude towards mathematics using LP word tasks due to an intervention administered to the experimental group. Further statistical treatment were performed to confirm this claim. The preliminary findings showed that students generally held negative attitude towards learning LP word tasks. These findings are consistent with other empirical research findings (e.g., Awofala, 2014).

**Table 2** shows the model representing the relationship between active learning heuristic problem solving approach and students' ATLPWTs. From **Table 2**, there exists a significant relationship and the model is explained by approximately 10% of variation within the



**Table 3.** ANOVA for the relationship between ALHPS approach & students' ATLPWTs

| Model |            | Sum of squares | df  | Mean square | F      | Sig.              |
|-------|------------|----------------|-----|-------------|--------|-------------------|
| 1     | Regression | 13.383         | 1   | 13.383      | 65.795 | .000 <sup>b</sup> |
|       | Residual   | 123.259        | 606 | .203        |        |                   |
|       | Total      | 136.642        | 607 |             |        |                   |

Note. <sup>a</sup> Dependent variable: attitude towards LP mathematics word tasks (ATLPWTs) & <sup>b</sup> Predictors: (Constant), active learning heuristic problem-solving approach (ALHPSA)

**Table 4.** A model representing the relationship between attitude & active learning

|                 | Coefficient | Standard error | t        | p-value | LLCI     | USCI     |
|-----------------|-------------|----------------|----------|---------|----------|----------|
| Constant        | -43.9672    | 4.1202         | -10.6712 | .0000   | -52.0588 | -35.8757 |
| Attitude        | 17.0392     | 1.1571         | 14.7263  | .0000   | 14.7669  | 19.3116  |
| Active learning | 16.2792     | 1.1384         | 14.3005  | .0000   | 14.0436  | 18.5148  |

data. The residual plots show that data is fairly normally distributed.

**Table 3** shows the significant positive relationship between ALHPSA and students' ALPWTs.

**Table 4** shows that keeping other variables constant, a unit change in the application of ALHPSA approach increases students' ATLPWTs by 0.31 (31%). Important to note is that this relationship is significant at 0.05% (95% confidence level). The findings show that both AL and attitude were significant within the lower limit confidence interval (LLCI) and upper limit confidence interval (ULCI) for attitude (14.7669, 19.3116) and active learning (14.0436, 18.5148) respectively.

## DISCUSSIONS, CONCLUSION, RECOMMENDATIONS, & LIMITATIONS

This study sought to investigate the effect of ALHPSA on students' ATLPWTs. The data were collected from 608 grade 11 students from eight secondary schools in eastern Uganda and central Uganda. The study findings revealed a direct significant positive relationship between ALHPSA and students' ATLPWTs. The psychometric properties of the adapted students' ATLPWTs and the ALHPSA instruments were found acceptable. This indicates that the ALHPSA enhanced students' attitude toward learning LP word tasks. The findings concord with previous empirical studies on the application of active learning strategies for improving and sustaining the learning of mathematics word tasks, and mathematics in general. The study findings further concord with the constructivism theoretical framework suggesting that educators should always review previous concepts and use them to construct new knowledge. The results of this study are likely to inform educational stakeholders in assessing students' ATLPWTs and provide remediation and interventional strategies aimed at creating a conceptual change in students' attitude towards solving LP word tasks and mathematics generally. This will further act as a lens in examining the relationships between students' achievement and their attitude towards learning specific mathematics concepts (content), as indicators of

students' confidence, motivation, usefulness, and enjoyment in learning LP word problems and mathematics in general.

The study findings show that students generally had negative attitude towards solving LP word problems. Although some students' ratings were below the neutral attitude (3), they indicated the usefulness of LP in daily lives. The experimental group showed a slightly favorable attitude towards LP word problems after an intervention because the problem-solving heuristic instruction was effectively applied during instruction as compared to students in comparison group who learned LP conventionally. Some students and teachers revealed that LP concepts are more stimulating, require students' prior conceptual knowledge and understanding of equations and inequalities, and are not interesting to learn just like other mathematics topics. The explanation provided indicated that some teachers do not apply suitable instructional techniques and learning materials to fully explain the concepts. However, it was observed that teachers encouraged students to constantly practice model formation from word problems to demystify the negative belief that LP is a hard topic, thereby encouraging them to understand LP and related concepts. Generally, students' ATLPWTs from the experimental group improved compared to their counterparts from the comparison group who almost had similar attitude towards solving LP word tasks before and after an intervention.

Both the experimental group and the control group acknowledged the fact that LP is a challenging topic (see **Appendix B**), although they highly recognized its significance in constructing models, and for optimization in real life. The importance of LP mainly rests in its application and thus teachers were tasked to help learners to develop a positive attitude, and conceptual understanding so that they can reason insightfully, think logically, critically and coherently. The teachers' competence in applying instructional strategies helped learners from the experimental group to gain deeper and broader insight, conceptual and procedural understanding, reasoning, and positive attitude. The control group in their conventional

instruction, however, still perceived LP as one of the hardest topics. The completed ATMI-SF questionnaires revealed that most students had a negative attitude towards solving LP word tasks. The teachers observed that by guiding students to work hard, and application of prior conceptual knowledge and understanding may favorably develop a positive attitude towards and performance in mathematics word tasks and LP in particular. Generally, students seemed not to have adequately developed the knowledge of logical thinking and reasoning. They did not view the learning of LP from a broader perspective beyond passing national examinations at Uganda Certificate of Education.

The results of this study point to important issues to the educational stakeholders in cultivating an early positive attitude in mathematics aimed at investigating specific content (topics) from primary school mathematics and beyond. This may be a potential strategy for applying different heuristic problem-solving approaches and strategies to significantly improve students' attitudes and consequently their performance. In this study, the problem-solving heuristic method supported collaboration and discussions between teachers and amongst students during the learning process. The students from the experimental group worked collaboratively in their small groups. The students helped and guided each other hence boosting their attitude and performance. Asempapa (2022) notes that teachers should always vary classroom instructional strategies taking into account individual students' academic and gender differences. This may help to change students' attitude towards solving LP word tasks, thereby providing both academic and social support.

Due to the application of ALHPSA, the low performing and average students greatly gained conceptual understanding and also acquired problem-solving skills, hence positive attitude. This generally motivated and enhanced students' learning and their attitude towards mathematics and solving LP word tasks in particular. Besides, the problem-solving heuristic approach applied to the experimental group boosted students' confidence in answering both routine and non-routine LP word problems. The findings further show that students' fear in comprehending LP word problems, attempting to solve LP tasks decreased. Thus, students' negative attitude towards solving LP word tasks changed. Instead, students were actively involved in problem-solving. This gradually built students' competence and confidence in solving LP word problems and related concepts which significantly fostered their positive attitude towards LP.

## CONCLUSION AND FUTURE RESEARCH DIRECTIONS

The purpose of this research was to explore the direct effect of the heuristic problem-solving approach on students' attitude towards mathematics, using LP word tasks. The findings show that the heuristic problem-solving approach positively and significantly changed students' attitude towards solving LP word tasks. The results of this study provide preliminary insights into the fundamental concepts of an introduction to LP concepts for learning advanced mathematics. Students' attitudes point to issues related to the demographic variables and latent constructs for learning mathematics. To gain more insight, we recommend that future researchers should use qualitative methods such as interviews and observation to provide more evidence on students' experiences in learning LP word tasks. The teachers' attitude towards LP is also a potential area for further investigation aimed at improving classroom instructional strategies. To achieve this, the teachers' in-service training professional development (PD) programs should be enacted to emphasize content knowledge and pedagogical content knowledge. Teachers coming together to share learning experiences and strategies, may help students to improve the learning of presumably "difficult topics" including LP word problems and mathematics generally. Indeed, teachers need routine PD support to successfully implement the learning strategies. The relationship between students' and teachers' demographic factors on students' ATLPWTs is another potential area for further investigation.

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**Data sharing statement:** Data supporting the findings and conclusions are available upon request from the corresponding author.

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**APPENDIX A**

**Table A1.** Questionnaire for students on application of active learning heuristic problem-solving

| No  | Item   | SD | D | N | A | SA |
|-----|--|----|---|---|---|----|
| 1.  | Teachers effectively applied GeoGebra software to demonstrate the relationship between equations and inequalities during the learning of linear programming. |    |   |   |   |    |
| 2.  | Teachers effectively applied graph papers during the learning of linear programming.   |    |   |   |   |    |
| 3.  | Teachers effectively applied grid boards during the learning of linear programming to help us understand all concepts.                                       |    |   |   |   |    |
| 4.  | I am able to comprehend the given linear programming word task.  |    |   |   |   |    |
| 5.  | I am able to devise a plan before answering any linear programming task.   |    |   |   |   |    |
| 6.  | I am able to carry out a plan before finding the solution to any linear programming word task.   |    |   |   |   |    |
| 7.  | I am able to look back and verify procedures to my solutions after answering tasks on linear programming word problems.                                      |    |   |   |   |    |
| 8.  | I am able to adequately read linear programming word tasks.  |    |   |   |   |    |
| 9.  | I am able to understand all procedures for finding solution of linear programming word problems.   |    |   |   |   |    |
| 10. | I am always able to transform linear programming word problems into models (equations & inequalities).   |    |   |   |   |    |
| 11. | I am able to process and adequately apply necessary skills needed for finding solutions to linear programming word problems.                                 |    |   |   |   |    |
| 12. | I am able to encode (understand and convert) concepts for finding the solution of linear programming word problems.  |    |   |   |   |    |



## APPENDIX B

## Sample Linear Programming Word Problems

1. **Q1.** A factory makes two kinds of bottle tops “Coca-Cola” and “Pepsi” tops. The same equipment can be used to make either. In making Coca-Cola tops, one man can supervise 10 machines and this batch will give a profit of pounds sterling (£) 50 per week. Pepsi cola tops yield a profit of (£) 250 a week, using 25 machines and 8 men. There are 200 machines and 40 men available. By taking  $x$  batches of Coca-Cola tops and  $y$  batches of Pepsi cola tops; write down inequalities for the:

- number of machines used,
- number of men employed, and
- expression for profit,  $P$ .

Use these inequalities to draw a suitable graph showing the region which satisfies them. From your graph, determine the numbers of Coca-Cola and Pepsi tops, which should be made to obtain the maximum profit. Hence find the maximum profit.

2. **Q2.** A wildlife club in a certain school wishes to go for an excursion to a national park. The club has hired a mini-bus and a bus to take students. Each trip for the bus is Shs.50,000 and that of a mini-bus is Shs.30,000. The bus has a capacity of 54 students and the minibus, 18 students. The maximum number of students allowed to go for the excursion is 216. The number of trips the bus makes do not have to exceed those made by the mini-bus. The club has mobilized as much as Shs.300,000 for transportation of the students. If  $x$  and  $y$  represent the number of trips made by the bus and mini-bus, respectively,

- Write down five inequalities representing the above information.
- Plot these inequalities on the same axes.
- By shading, unwanted regions show the region satisfying the above inequalities.
- List the possible number of trips each vehicle can make.
- State the greatest number of students who went for the excursion.

3. **Q3.** A private car park is designed in such a way that it can accommodate  $x$  pickups and  $y$  minibuses at any given time. Each pickup is allowed 15 m<sup>2</sup> of space and each mini-bus 5 m<sup>2</sup> of space. There is only 400 m<sup>2</sup> of space available for parking. Not more than 35 vehicles are allowed in the park at a time. Both types of vehicles are allowed in the park. But at most 10 mini-buses are allowed at that time.

- Write down all the inequalities to represent the above information.
  - On the same axes plot the graphs to represent the above inequalities in (i) hence shade out unwanted regions.
- If the parking charges for pick up are Shs.500 and that for a minibus is Shs.800 per day, find how many vehicles of each type should be parked to obtain maximum income, hence find the daily maximum income.

**Table B1.** Farmer’s estimates for the project

|                         | Carrots     | Potatoes    |
|-------------------------|-------------|-------------|
| Harvesting cost @ ha    | Shs.95,000  | Shs.60,000  |
| Number of working hours | 12 days     | 4 days      |
| Expected profit per ha  | Shs.228,000 | Shs.157,000 |

4. **Q4.** A farmer plans to plant an 18hectare field with carrots and potatoes. The farmer’s estimates for the project are shown in **Table B1**.

The farmer has shs.1,140,000 only to invest in the project. The total number of working days is 120. By letting  $x$  represent the number of hectares to be planted with carrots and  $y$  the number of hectares to be planted with potatoes

- Write down inequalities for
  - cost of the project,
  - working days,
  - number of hectares used in the project, and
  - the possibility that the field will at least be used for planting either carrots or potatoes.
- Write down an expression for the profit  $P$  in terms of  $x$  and  $y$ .
-

- (i) On the same axes, plot graphs of the inequalities in (a) and (b) by shading out the unwanted regions.
- (ii) Use your graph to determine how the farmer should use the field to maximize profit. Hence, find the farmer's maximum profit.
5. **Q5.** At a graduation party, the guests are served beer and soda. At least twice as many crates of beer as crates of soda are needed. A crate of beer contains 25 bottles, and a crate of soda contains 24 bottles. More than 200 bottles of beer and soda are needed. A maximum of Shs.500,000 may be spent on beer and soda. Assume a crate of beer costs shs.40,000 and that of soda costcost.15,000.
- (a)
- (i) Form inequalities to represent the above information.
- (ii) Represent the above inequalities on the same axes.
- (iii) By shading the unwanted regions, represent the region satisfying the inequalities in (a) (i) above.
- (b) From your graph, find the number of crates of beer and soda that should be bought if the cost is to be as low as possible. Find the amount that was paid for those crates of beer and soda.
6. **Q6.** A bicycle factory assembles two types of bicycles, Road master and Hero on different assembly lines. An assembly line for the road masters occupies an area of  $60\text{m}^2$  of the floor space. The floor space available for all the assembly lines is  $420\text{m}^2$ . The assembly line of the road master needs 10 men to operate it and that of the hero needs 16 men to operate it. The assembly lines need a maximum of 120 men to operate them.
- (a) If  $x$  and  $y$  represent the number of assembly lines for road master and hero, respectively.
- (i) Form four inequalities to represent the given information.
- (ii) Draw graphs on the same axes to represent the inequalities in (a) (i) above. Shade the unwanted regions.
7. **Q7.** A private car park is designed in such a way that it can accommodate  $x$  pickups and  $y$  minibuses at any given time. Each pickup is allowed  $15\text{m}^2$  of space and each miminibus  $5\text{m}^2$  of space. There is only  $400\text{m}^2$  of space available for parking. Not more than 35 vehicles are allowed in the park at a time. Both types of vehicles are allowed in the park. But at most 10 mini-buses are allowed at a time.
- (a)
- (i) Write down all the inequalities to represent the above information.
- (ii) On the same axes plot the graphs to represent the above inequalities in (i) hence shade out unwanted regions.
- (b) If the parking charges for pick up are Shs.500 and that for a minibus is Shs.800 per day, find how many vehicles of each type should be parked to obtain maximum income, hence find the daily maximum income.

**APPENDIX C**

**Table C1.** ATMI-SF questionnaire for students’ attitude towards linear programming word problems

| No  | Item  | SD | D | N | A | SA |
|-----|---|----|---|---|---|----|
| 1.  | Linear programming word problems are a very worthwhile and necessary topic in mathematics.  |    |   |   |   |    |
| 2.  | Linear programming is one of the most important topics in mathematics for students to study.  |    |   |   |   |    |
| 3.  | Linear programming lessons are very helpful no matter what I decide to study in the future.   |    |   |   |   |    |
| 4.  | Studying linear programming is important in everyday life.  |    |   |   |   |    |
| 5.  | Learning linear programming makes me feel nervous.  |    |   |   |   |    |
| 6.  | I am always under a terrible strain in a mathematics class involving the learning of linear programming word problems.                  |    |   |   |   |    |
| 7.  | I am always confused in my mathematics class whenever we are learning linear programming word problems.                                 |    |   |   |   |    |
| 8.  | I feel a sense of insecurity and nervousness when solving a linear programming word problem.  |    |   |   |   |    |
| 9.  | I have usually enjoyed studying & solving linear programming & related word problems in school.   |    |   |   |   |    |
| 10. | Linear programming word problems is one of the interesting topics in mathematics.   |    |   |   |   |    |
| 11. | I am always happy, and like learning and solving new tasks in linear programming word problems.   |    |   |   |   |    |
| 12. | I am willing to solve & take more than the required amount of linear programming word problems.   |    |   |   |   |    |
| 13. | The challenge of learning linear programming word problems appeals to me.   |    |   |   |   |    |
| 14. | I plan & am confident to take as many linear programming word problems as possible in advanced mathematics.                             |    |   |   |   |    |
| 15. | Questions on linear programming should not be made compulsory during national examinations.<br>Please select “agree” for this question. |    |   |   |   |    |

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