Indian J Phys
https://doi.org/10.1007/s12648-018-1169-5

ORIGINAL PAPER

Dust ion acoustic double layers in a 4-component dusty plasma

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Received: 05 July 2017 / Accepted: 04 December 2017

Abstract: In this paper, we study the Dust ion acoustic (DIA) solitons in an unmagnetised dusty plasma comprising of cold dust particles, electrons that follow Cairns distribution, warm inertial ions, and ion-beams of equal mass, using arbitrary amplitude technique. Our results show that it is possible for both rarefactive (negative) and compressive (positive) DIA solitary waves to coexist. Interestingly, double layers could not limit the existence of solitary waves. These results can therefore help to understand the mechanism for decelerating protons in the accretion flow onto neutron stars in a binary system at radial distances where the effect of magnetic field can be neglected.

Keywords: Solitons; Dusty plasma; Double layers

PACS Nos.: 52.35.Sb - Solitons BGK modes; 52.27.Lw - Dusty or complex plasmas; plasma crystals

1. Introduction

Over the years, dusty plasmas have received considerable attention from several authors [1–6]. This is because of their ubiquity in the universe [7] and a rich linear and nonlinear wave phenomena [8].

The first theoretical study on the existence of low frequency dust ion acoustic solitary waves (DIASWs) was conducted by Shukla and Silin in 1992 [1]. Since then, most studies on nonlinear dusty plasma waves have considered unmagnetised plasma models. In the case of the four component dusty plasma models, it is possible to obtain either rarefactive or compressive double layers. In the model by [2], existence of large amplitude electrostatic double layers in four-component collisionless and unmagnetised dusty plasma was investigated. The species considered were, electrons, two distinct positive ion species of different temperatures, and extremely heavy negatively charged dust particles and only compressive double layers were possible.

Later, [3] studied large amplitude double layers in a dusty plasma with an arbitrary streaming ion beam and found that both the temperature of dust and ion beam temperature play significant roles in determining the region of double layers. Therefore, the ion-beam plays a significant role in the formation of double layers.

Additionally, [4] studied DIASWs in a plasma with kappa-distributed electrons. They showed that negative dust supports solitons of both polarities while positive dust supports only positive potential solitons. Recently, [5] also studied arbitrary amplitude DIASWs and double layers in a kappa distributed electron plasma, and provided existence domains of arbitrary amplitude rarefactive double layers.

In this study, such structures as compressive and rarefactive double layers as well as solitons will be investigated for an electron population that follow Cairns distribution.

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Published online: 22 February 2018

2. Plasma model and governing equations

We study the model formulation for a 4-component collisionless and unmagnetised dusty plasma where the dust grains are negatively charged and stationary, while the electrons are Cairns distributed and the positively charged particles are warm inertial ions and ion-beams of equal mass.

The Cairns distribution by [9] gives the density of nonthermally distributed electrons $N_e(\varphi)$ as:

$$N_e(\varphi) = N_{e0}\left[1 - \beta\left(\frac{e\varphi}{k_B T_e}\right) + \beta\left(\frac{e\varphi}{k_B T_e}\right)^2\right]\exp\left[\frac{e\varphi}{\mathrm{k_B T_e}}\right], \tag{1}$$

where N_{e0} is the equilibrium density of electrons, e is the electronic charge, φ is the electrostatic potential, k_B is the Boltzmann constant, T_e is the electron temperature and β is the nonthermal parameter which measures the deviation from Maxwellian equilibrium [6].

The un-normalized fluid equations for warm inertial ions and ion-beam (in the prescence of pressure effects) are, continuity, momentum and pressure equations respectively as:

$$\frac{\partial N_j}{\partial t'} + \frac{\partial}{\partial x'}(N_j V_j) = 0, \tag{2}$$

$$\frac{\partial V_j}{\partial t'} + V_j\frac{\partial V_j}{\partial x'} + \frac{q_j}{m_j}\frac{\partial \varphi}{\partial x'} + \frac{1}{N_j m_j}\frac{\partial P_j}{\partial x'} = 0, \tag{3}$$

$$\frac{\partial P_j}{\partial t'} + V_j\frac{\partial P_j}{\partial x'} + 3P_j\frac{\partial V_j}{\partial x'} = 0, \tag{4}$$

and these three equations can be coupled through Poissons equation,

$$\epsilon_0\frac{\partial^2\varphi}{\partial x'^2} + \sum_j q_j N_j = 0, \tag{5}$$

where, V_j, N_j and $\mathrm{P_j}$ are the un-normalized ion or ion beam fluid speeds, number density, and thermal pressure for the jth species respectively; $m_j(q_j)$ is the ion or ion beam mass (charge), ϵ_0 is the permittivity of free space and $x'(t')$ is the un-normalized space (time) variable. The assumption of charge neutrality at equilibrium yields,

$$\sum_j q_j N_{j0} = 0, \tag{6}$$

where the index "0" denotes the equilibrium charge density values. In order to normalize, it is convinient to introduce the dimensionless quantities (x, t, ϕ, n, v) for the parametric space which limit the existence of solitons and double layers. Here x and t are the normalized space and time variables, while ϕ, n, v are the normalized potential, number density, ion or ion beam fluid speeds, respectively.

The normalized system of Eqs. (2)–(4) are:

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j u_j) = 0, \tag{7}$$

$$\frac{\partial u_j}{\partial t} + u_j\frac{\partial u_j}{\partial x} + \frac{\sigma_j}{n_j}\frac{\partial p_j}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \tag{8}$$

$$\frac{\partial p_j}{\partial t} + u_j\frac{\partial p_j}{\partial x} + 3p_j\frac{\partial u_j}{\partial x} = 0. \tag{9}$$

The independent variables, x and t, are normalized to a mixed electron-ion effective Debye length $\lambda_{Deff} = (\varepsilon_0 k_B T_e/N_{i0}e^2)^{1/2}$ and the inverse ion plasma frequency $\omega_{pi}^{-1} = (N_{i0}e^2/\varepsilon_0 m_i)^{-1/2}$, respectively; the dependent variables, u_j, n_j, and p_j, are normalized to the ion acoustic speed $C_s = (k_B T_e/m_i)^{1/2}$, the ion density N_{i0}, and the ion pressure $P_{i0} = N_{i0}k_B T_i$, respectively.

In transforming to the stationary frame, it is convinient to let $x = \xi - Mt$ implying that $\partial/\partial x = \partial/\partial\xi$ and $-M\partial/\partial\xi = \partial/\partial t$ and then integrating with respect to ξ using the boundary conditions. As $|\xi| \to \infty$; $p_j \to 1$, $\phi \to 0$, $n_j \to f_j$ i.e. $\mathrm{n_i} \to 1$ and $\mathrm{n_b} \to \mathrm{f_b}$, while $u_j \to u_{j0}$ i.e. $\mathrm{u_{i0}} \to 0$ and $\mathrm{u_{b0}} \to \mathrm{u_b}$.

The electron, ion and ion beam densities are obtained repectively as:

$$n_e(\phi) = (1 - \beta\phi + \beta\phi^2)\exp^{\phi}. \tag{10}$$

and

$$n_j^2 = \frac{\varpi}{6\sigma_j} \pm \frac{\sqrt{\varpi^2 - 12\sigma_j f_j^5 (M - u_{b0})^2}}{6\sigma_j}, \tag{11}$$

where $\varpi = f_j^2[f_j(M - u_{b0})^2 + 3\sigma_j - 2\phi f_j]$.

By setting $n_j = \sqrt{p} \pm \sqrt{q}$ it implies that

$$n_j^2 = (p + q) \pm 2\sqrt{pq}. \tag{12}$$

Comparing Eqs. (11) and (12), we obtain

$$p + q = \frac{\varpi}{6\sigma_j}, \tag{13}$$

and

$$p - q = (M - u_{b0}) f_j^2 \sqrt{\frac{f_j}{3\sigma_j}}. \tag{14}$$

Solving Eqs. (13) and (14) simultaneously, we have the density equation for both ion and ion beam as:

$$n_j = \frac{f_j^2}{\sqrt{12\sigma_j f_j}}\left\{\left[\left(M - u_{j0} + \sqrt{\frac{3\sigma_j}{f_j}}\right)^2 - 2\phi\right]^{\frac{1}{2}} \pm \left[\left(M - u_{j0} - \sqrt{\frac{3\sigma_j}{f_j}}\right)^2 - 2\phi\right]^{\frac{1}{2}}\right\}. \tag{15}$$

A similar density equation has been recently derived by [10] in a three component plasma without ion beam. The ±

sign are used for supersonic and subsonic species respectively [11, 12]. Following the working of [10], the choice of the sign to be used in this model must satisfy the boundary condition, $n_j(\phi = 0) \to f_j$. Therefore, it was the negative sign that satisfies the conditions and has a physical relevance in this model. Subsequently, the ion or ion beam density is,

$$n_j(\phi) = \frac{f_j^2}{\sqrt{12\sigma_j f_j}} \left\{ \left[\left(M - u_{j0} + \sqrt{\frac{3\sigma_j}{f_j}} \right)^2 - 2\phi \right]^{1/2} - \left[\left(M - u_{j0} - \sqrt{\frac{3\sigma_j}{f_j}} \right)^2 - 2\phi \right]^{1/2} \right\}. \quad (16)$$

It can be observed from Eq. (16) that for $\phi = \phi_{l1} = \frac{1}{2}\left(M - u_{j0} + \sqrt{\frac{3\sigma_j}{f_j}}\right)^2$, n_j becomes complex and for $\phi = \phi_{l2} = \frac{1}{2}\left(M - u_{j0} - \sqrt{\frac{3\sigma_j}{f_j}}\right)^2$, n_j becomes finite. Thus, the limiting potentials to the existence of solitary waves are $\phi \geq \phi_{l1}$, and $\phi > \phi_{l2}$. For ions i.e. ($j = i$), the density ratio, $f_j = f_i = N_{i0}/N_{i0} = 1$, and the uniform warm inertial ions equilibrium speed, $u_{j0} = u_{i0} = 0$, and it gave

$$n_i(\phi) = \frac{1}{\sqrt{12\sigma_i}} \left\{ \left[A_i^+ - 2\phi\right]^{1/2} - \left[A_i^- - 2\phi\right]^{1/2} \right\}. \quad (17)$$

where $A_i^{\pm} = \left(M \pm \sqrt{3\sigma_i}\right)^2$. For ion beam i.e. ($j = b$), the density ratio $f_j = f_b = N_{b0}/N_{i0}$, and the energetic warm inertial ion beams equilibrium speed, $u_{j0} = u_{b0}$, and it was obtained to be,

$$n_b(\phi) = \frac{f_b^2}{\sqrt{12\sigma_b f_b}} \left\{ \left[A_b^+ - 2\phi\right]^{1/2} - \left[A_b^- - 2\phi\right]^{1/2} \right\}. \quad (18)$$

where $A_b^{\pm} = \left(M - u_{b0} \pm \sqrt{\frac{3\sigma_b}{f_b}}\right)^2$. Now that we have the densities of all species, the Poisson's equation (Eq. 5) is used to obtain the required pseudo potential.

2.1. Theory of Sagdeev pseudo potential for a four component dusty plasma model

The normalized form of Eq. (5) looks like,

$$\frac{\partial^2 \phi}{\partial x^2} = n_i(\phi) + n_b(\phi) - n_e(\phi) - Z_d \frac{N_{d0}}{N_{i0}}. \quad (19)$$

Eq. (19) can be transformed as follows:

$$\frac{d^2\phi}{d\xi^2} = -[n_e(\phi) + (1 + f_b - f_e) - n_i(\phi) - n_b(\phi)] = G(\phi, M), \quad (20)$$

where $Z_d \frac{N_{d0}}{N_{i0}} = 1 + \frac{N_{b0}}{N_{i0}} - \frac{N_{e0}}{N_{i0}} = (1 + f_b - f_e)$. The integral of $G(\phi, M)$ can be expressed as:

$$\int_0^{\phi} G(\phi, M) d\phi = \int_0^{\phi} \frac{1}{2} \frac{d}{d\phi} \left(\frac{d\phi}{d\xi}\right)^2 d\phi = \frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2. \quad (21)$$

Therefore, Eq. (20) yields

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + S(\phi, M) = 0, \quad (22)$$

where $S(\phi, M) = -\int_0^{\phi} G(\phi, M) d\phi$, which governs the soliton behaviour. Equation (22) resembles an energy integral in classical mechanics for a particle with unit mass in a conservative force field with ϕ being the particle pseudoposition and ξ playing the role of pseudotime [13].

Integrating Eq. (20) with respect to ϕ the Sagdeev potential or pseudo potential, $S(\phi, M)$ is obtained as:

$$\begin{aligned} S(\phi, M) = {} & (f_e - f_b - 1)\phi \\ & + f_e[1 + 3\beta - (1 + 3\beta - 3\beta\phi + \beta\phi^2)e^{\phi}] \\ & + \frac{1}{\sqrt{108\sigma_i}} \left\{ \left[A_i^+ - 2\phi\right]^{\frac{3}{2}} - \left[A_i^- - 2\phi\right]^{\frac{3}{2}} \right\} \\ & + \frac{f_b^2}{\sqrt{108\sigma_b f_b}} \left\{ \left[A_b^+ - 2\phi\right]^{\frac{3}{2}} - \left[A_b^- - 2\phi\right]^{\frac{3}{2}} \right\} \\ & + (M^2 + \sigma_i) + f_b(M - u_{b0})^2 + \sigma_b. \end{aligned} \quad (23)$$

This potential equation is very helpful in studying the behaviour of nonlinear waves [14].

3. Results and discussion

3.1. Existence regions

Solitary waves can only propagate when the following conditions are fulfilled.

1. At $\phi = 0$ (the origin);
   (a) $S(\phi, M) = S'(\phi, M) = 0$, which represents the requirement that both the electric field and the charge density be zero far from the localized solitary structure.
   (b) $S''(\phi, M) < 0$, such that the origin is unstable the sign of the derivative of the charge density is compatible with the sign of the electric field at

large distances, see Eq. (22) i.e. has a maximum at the origin.

2. At $\phi = \phi_m$ (the amplitude of SWs and DLs);

   (c) $S'(\phi_m, M) < (>) 0$ for some $\phi_m < (>) 0$,
   (d) $S(\phi, M) < 0$ for $0 < |\phi| < |\phi_m|$ for some M.

where the primes in (a), (b) and (c) denote derivatives with respect to ϕ. Additionally, double layers, require that conditions (a)–(d) and $S(\phi_m, M) = S'(\phi_m, M) = 0$ hold for some M at $\phi_m \neq 0$.

Clearly, condition (a) is satisfied by Eq. (23). When condition (b), was applied, i.e. $S''(\phi = 0, M) < 0$, it yielded:

$$S''(\phi = 0, M) = -f_e(1-\beta) + \frac{1}{(M^2 - 3\sigma_i)} + \frac{f_b}{(M - u_{b0})^2 - 3\frac{\sigma_b}{f_b}} < 0. \tag{24}$$

This requirement constituted the soliton (existence) condition to be fulfilled, i.e. the root of $S''(\phi = 0, M) = F(M) = 0$ in terms of the Mach number M defined a critical value as a lower limit for M, say M_s in the (F, M) space.

$$-f_e(1-\beta) + \frac{1}{(M_s^2 - 3\sigma_i)} + \frac{f_b}{(M_s - u_{b0})^2 - 3\frac{\sigma_b}{f_b}} = 0, \tag{25}$$

We observed that, Eq. (25) is a quadratic equation in M_s and as a result there are four roots of M_s which are the critical acoustic velocities lying between four successive thermal velocities. Cleary, M_s depends on $f_e, f_b, \beta, u_{b0}, \sigma_i, \text{and} \sigma_b$ for a given set of parameters. Consequently, the solution of M_s take the shape shown in Fig. 1.

In this 4 component plasma model, there are four (4) acoustic existence ranges for solitary waves (Fig. 1).

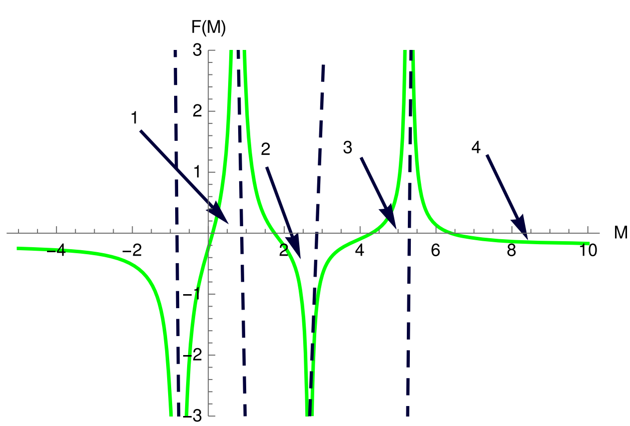

Fig. 1 Plot showing existence ranges for solitary waves in this model (Eq. 25) where $f_e = 0.4; f_b = 0.7; \beta = 0.5; \sigma_i = 0.2; \sigma_b = 0.4; u_{b0} = 4.0$

Existence domains for stationary solitary structures require that $M > M_s$ [15]. With the parameters used in Fig. 1, only region 3 and 4 satisfy condition 1(b) expressed in Eq. (24).

In ion beam driven dusty plasmas, three longitudinal electrostatic waves are found to propagate as an dust ion acoustic mode (DIA), fast (F) or slow (S) modes (refer to [6, 17] for a detailed numerical analysis). Both [6, 17] showed that $M_s > u_{b0}$ (from Eq. 25) in order to prevent ion-ion instabilty from occuring. This kind of instability has been confirmed in the laboratory setting by [19]. In our model, solitons (F-mode) exists for $M_s > u_{b0} \pm \sqrt{\frac{3\sigma_b}{f_b}}$, i.e. the ion beam concentration has to increase with respect to the plasma ion density and the ion beam speed has to be very small compared to the phase speed [18].

On the other hand, in the absence of ion beam $f_b = 0$,

$$M_s = \sqrt{\frac{1}{f_e(1-\beta)} + 3\sigma_i}. \tag{26}$$

In this case, the influence of nonthermal electrons on the soliton velocity threshold is that, as β or σ_i increases then, M_s, must increase while in the absence of nonthermal electrons i.e. $\beta = 0$,

$$M_s = \sqrt{\frac{1}{f_e} + 3\sigma_i}. \tag{27}$$

Therefore, for cold ions, we obtained:

$$M_s = \sqrt{\frac{1}{f_e}} = \sqrt{\frac{N_{i0}}{N_{e0}}}, \tag{28}$$

which is as originally found for DIA solitons by [1] in a plasma with cold ions and polytropic electrons [16] and for ion acoustic solitons in a two temperature ion plasma [20]. Thus, for an electron-ion plasma ($f_e = 1$), one recovers the usual lower Mach number limit $M_s = 1$.

Besides the lower limit on the soliton speed, soliton existence regions may be bounded by a number of other possible physical constraints. For instance, the occurrence of a double layer when one of the species reach a sonic point (resulting in infinite rarefaction or compression of the species), or a density assumed on a complex value [16, 20]. However, of recent, [21] found that it was still posssible to obtain solitons beyond double layers for some models.

In this model, electrons and negative dust densities would provide a limit for negative potential solitons and the ion beam or ion density would limit positive potential solitons. On close examination of electron density (1), it was found to be well behaved for all $\phi > 0$. As a result, the negative potential solitons were not limited by electron density and or dust. Moreover, as it was seen earlier , $n_j(\phi)$ became complex and so was $p_j(n_j)$, when $\phi \geq \phi_{j1}$ and $\phi > \phi_{j2}$ and the Sagdeev pseudo potential,

$S(\phi_{j\alpha}, M); \alpha; 1, 2$ was finite at that point. Thus, $S(\phi_{j\alpha}, M)$ satisfies the requirement for limiting potential, i.e. $S(\phi_{j\alpha}, M) \geq 0$, [16, 22, 23].

The limiting potential or the critical potential, ϕ_c of ϕ was such that $0 < \phi < \phi_c$, where $\phi_c = \mathrm{Min}\{\phi_{j\alpha}\} = \phi_{b2}$. Therefore, the practical limiting potential for this model was the ion beam contribution. Values that were accesible, for the Mach number were those for which the Sagdeev well yielded a root, ϕ_c before the complex n_j was reached, and hence the larger possible values of M were obtained by imposing the requirements $S(\phi_{j\alpha}) \geq 0$. The upper limit on the speed of the solitary waves (say M_u) was obtained from $S(\phi_{b2}, M) = 0$. The condition $S(\phi_{b2}, M) > 0$ led to an upper limit on f, i.e. $\phi_{b2} = \frac{1}{2} A_b^-$.

3.2. Positive potential solitons, negative potential solitons and double layers

Numerical analysis of Eq. (23), revealed existence of both compressive and rarefactive solitons on the profile of the electrostatic potential (Fig. 2). It can be seen that the amplitudes of solitons of either polarity increased with the Mach number, with the rarefactive type (Fig. 2 *Left Panel*) having larger amplitudes as compared to the compressive type (Fig. 2 *Right Panel*). Further, positive potential solitons were limited by the potential of ion beam for Mach numbers exceeding $M = 5.9$ for the given parameters. On the other hand, negative potential solitons could not be stopped by the existence of double layers (see Fig. 2 *Left Panel* for $M = 6.38949$). This result concurs with the conclusion of other previous studies [21].

Figure 3 presents the soliton solution corresponding to the Sagdeev potential shown in Fig. 2. For our choice of Mach numbers, $M = 5.85$ for the compressive solitons and $M = 6.75$ for the rarefactive solitons, we note that the compressive solitons have smaller amplitudes than rarefactive type. Finally, both the amplitude $\phi(\xi)$ and the width ξ of the soliton increase as Mach number M increases [12].

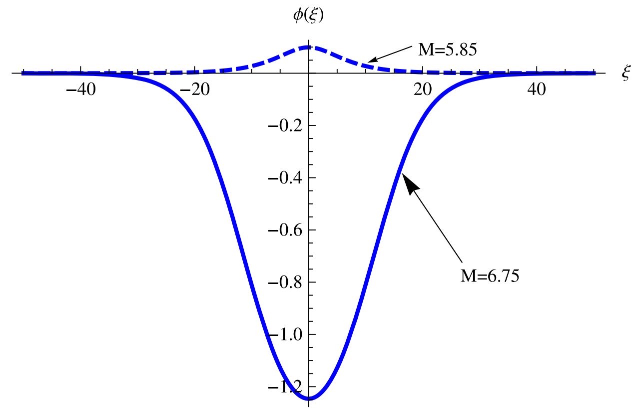

Fig. 3 Plot showing dependence of superimposed soliton solutions, corresponding to equation (23), for physical parameter variations as in Fig. 2

Double layers can result in trapping of ions and consequent loss of electrons energy [11]. [24], argues that DLs can result in the decelerating mechanism of ions present in accretion columns of a disc around a neutron star in a binary system. This result has the potential of explaining several astronomical observations that indicate the existence of double layers [25, 26].

4. Conclusions

In this paper, we have for the first time analysed the existence of DIAWs in a four (4) component dusty plasma with ions, Cairns distributed electrons, and a streaming ion beam. Both theoretical and numerical results are in conformity with previous results [16, 20, 21]. The amplitude of solitons of either polarity increased with increase in Mach number. The negative potential solitons emerged

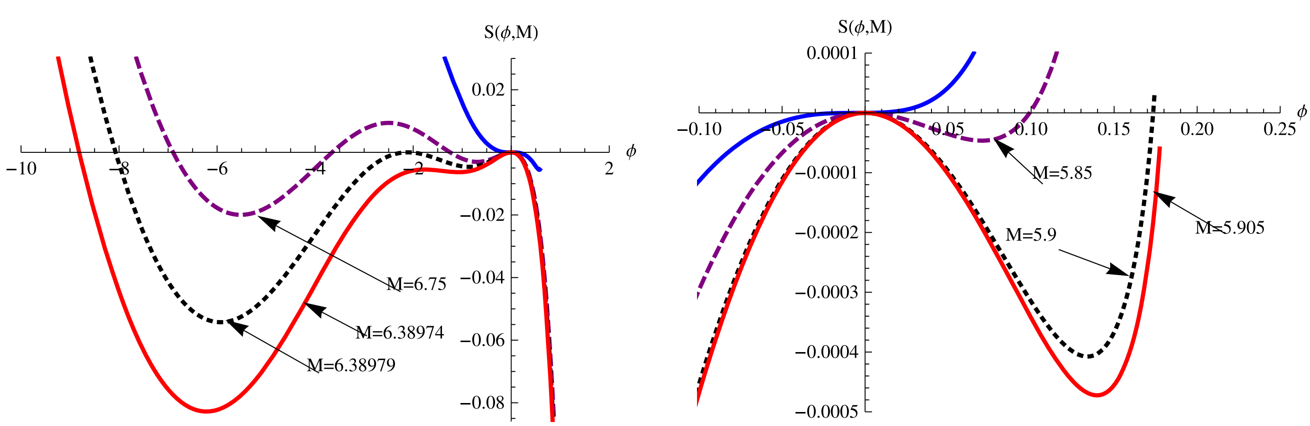

Fig. 2 Variation of Sagdeev potential $S(\phi, M)$ with electrostatic potential ϕ. The parameter values used are; *Left Panel* (*Right Panel*) f_b = 0.7(0.7); β = 0.5(0.5); σ_i = 0.2(0.2); σ_b = 0.4(0.4); u_{b0} = 4.0(4.0); and f_e= 0.1(0.9674)

unstopable by existence of double layers while the negative solitons were limited by the potential of ion beam for Mach numbers exceeding $M = 5.9$ for given parameters. This is an extraordinary and brilliant feature of ion beam driven dusty plasmas. These results can be applied in engineered materials, dielectrics, and can help explain the mechanism for decelarating protons in the accretion flow in astronomical environments where the effect of magnetic fields can be neglected.

Acknowledgements We are grateful to International Science Programme (ISP) for funding this study and Mbarara University of Science and Technology for providing the ambient environment for the studies.

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