# APPLYING THE SOLO TAXONOMY IN ASSESSING AND FOSTERING STUDENTS' MATHEMATICAL PROBLEM-SOLVING ABILITIES



3 authors:

Angel Mukuka
Mukuba University


Sudi Balimuttajjo
Mbarara University of Science & Technology (MUST)


Vedaste Mutarutinya
University of Rwanda

# APPLYING THE SOLO TAXONOMY IN ASSESSING AND FOSTERING STUDENTS' MATHEMATICAL PROBLEM-SOLVING ABILITIES

Angel Mukuka[1], Sudi Balimuttajjo[2], Vedaste Mutarutinya[1]
[1]University of Rwanda, [2]Mbarara University of Science and Technology, Uganda

*This paper examines how the Structure of the Observed Learning Outcomes (SOLO) taxonomy can influence assessment and instructional approaches in fostering students' mathematical problem-solving abilities. To answer this question, we employed a one-group pre- and post-test design involving 80 Grade 11 students from one intact class at a secondary school in Ndola district of Zambia. The test items administered on research participants followed the SOLO taxonomy response structure. On average, participants performed significantly better in the post-test* $(M = 53.75, SE = 2.34)$ *compared to pre-test* $(M = 32.0, SE = 1.99), t(79) = -9.796, p < .05, r = .74$. *Results further revealed that the SOLO taxonomy is a useful assessment tool in the sense that; (i) it provides direction on the needy areas of focus in responding to students' mathematical learning difficulties, (ii) it gives an indication on the possibility of remedial work/lessons, and (iii) it can lead to some pedagogical changes aimed at addressing students' learning difficulties. These results provide evidence of the usefulness of the SOLO taxonomy to influence assessment and instruction.*

## INTRODUCTION

Mathematics plays a very important role in developing learners' intellectual abilities in logical reasoning, critical and abstract thinking, and spatial visualisation. Mathematics does not only prepare students for further education and future career opportunities but equips them with tools for solving real-world problems. One of the aims of mathematics teaching in Zambian secondary schools (Curriculum Development Centre, 2013) is that of building up an appreciation of learned concepts so that the learner can apply them for problem-solving in everyday life. However, students' success in school mathematics in Zambia has been constrained by several factors some of which are to do with instructional and assessment approaches that teachers adopt. High failure rates, students' inadequate understanding and comprehension of mathematical concepts, and the backwash effects of examinations have been reported not only in Zambia (Examinations Council of Zambia, 2018) but other contexts as well (Abdullah, Abidin, & Ali, 2015; Bethell, 2016; Conn, 2017; Jurdak & Mouhayar, 2013; Mulbar, Rahman, & Ahmar, 2017).

Additionally, there is a multitude of evidence that teachers from different contexts and settings (Aksit, Niemi, & Nevgi, 2016; Kartina et al., 2011; Zakaria & Iksan, 2007) have continued their hold to traditional forms of assessment and instruction. A common mode of instruction in such classrooms is one in which the teacher lectures, and the students listen. Thereafter, learners' conceptual understanding is assessed through a class exercise or a test where they are expected to answer routine questions by mimicking examples given to them during the lesson. In such kinds of environments, there is little emphasis on non-routine tasks (Mueller, Yankelewitz, & Maher, 2014; Mulbar et al., 2017) that could trigger higher-order thinking among learners because most of the tasks given to students require them to memorise previously learned facts. We believe that routine tasks provide little insights into the processes underlying the acquisition of relevant mathematical skills and



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competencies. Therefore, teachers ought to take action on how they can improve their instructional and assessment approaches to create meaningful opportunities for learners to learn.

With this background, this paper aims to highlight the application of the Structure of the Observed Learning Outcomes (SOLO) taxonomy in assessing and fostering students' mathematical problem-solving abilities. Our analysis was guided by the following research question:

> How does the SOLO Taxonomy influence assessment and instructional approaches in fostering students' mathematical problem-solving abilities?

By providing answers to the above-stated question, it was anticipated that teachers would be informed on how they can utilise the SOLO taxonomy to improve their assessment and instruction, and eventually improve their students' mathematical problem-solving abilities.

## THEORETICAL PERSPECTIVES AND BRIEF REVIEW OF LITERATURE

Assessment plays a significant role in fostering students' mathematical problem-solving abilities in the sense that it provides an opportunity for a teacher to understand what students can do and how they can do it. This allows a teacher to use such information to modify instruction. The National Council of Teachers of Mathematics (2000) has recommended instructional approaches that are designed to enable all students of school mathematics to gain new knowledge through problem-solving activities. However, assessment of students' mathematical problem-solving abilities has been inadequate if not absent in most mathematics classrooms. This is why Lim and Wun (2010) called for a revision and an improvement of assessment procedures to provide useful information for instructional reform. Informed by previous studies (e.g. Collis, Romberg, & Jurdak, 1986; Jurdak & Mouhayar, 2013; Lim & Wun, 2010, 2012; Mulbar et al., 2017), we anticipated that the SOLO taxonomy can provide teachers with insights into relevant instructional and assessment modifications in fostering students' mathematical problem-solving abilities.

The SOLO taxonomy was developed by Biggs and Collis (1982). Based on their initial analysis of responses from various sources, they realised that they were actually dealing with two distinct phenomena. The first one was closely related to Piaget's stages of cognitive development and they referred to it as the Hypothetical Cognitive Structure (HCS). The latter was to do with the structure of any given response without necessarily associating it with a particular stage of intellectual development. They referred to it as the Structure of Learned Responses or Outcomes (SOLO). According to Collis et al. (1986), the SOLO taxonomy forms a theoretical basis for developing a technique that can be used to assess reasoning in mathematical problem-solving activities.

Biggs and Collis (1982) theorised that student responses to a task or question can be categorised into five levels ranging from pre-structural to extended abstract as itemised below:

i. Pre-structural: responses depicting inappropriate or no understanding of a question /task at hand

ii. Unistructural: responses representing only one relevant aspect of the task

iii. Multi-structural: responses showing several relevant aspects that are disjoint or independent

iv. Relational: responses that merge all relevant aspects of information into structure formation and generalisation

v. Extended abstract: responses depicting the application of the existing or emerging structure into a new and more abstract situation such as new topic or subject area.

According to Jurdak and Mouhayar (2013), the above SOLO levels occur within each of Piaget's four-stage theory of cognitive development and they are hierarchically structured. This means that the SOLO levels from pre-structural to the extended abstract represent an increase in both the use of relevant information and the complexity of the response structure. Scholars like Eggen and Kauchak (2000) have criticised Piaget's four-stage theory of cognitive development because of its emphasis on the learner's age. It has been argued that students may not always operate at the level corresponding to their age. This is why it is common for learners to operate at one level in relation to a particular concept but at a different level in relation to another concept. For that reason, the SOLO taxonomy offers a good alternative in the sense that the five levels of students' response structure are assessed on the basis of the task at hand.

Several studies (e.g. Collis et al., 1986; Jurdak & Mouhayar, 2013; Lim & Wun, 2010, 2012) have applied the SOLO taxonomy in describing students' developmental stages of algebraic solving ability. They combined the SOLO technique (Biggs & Collis, 1982) and the idea of super-items (Cureton, 1965) to design questions or items whose responses would indicate the cognitive ability at a certain level as reflected in the SOLO structure of that particular question. To our knowledge, there has been little emphasis in literature on how the SOLO taxonomy can influence instructional and assessment practices in fostering students' mathematical problem-solving abilities. For that reason, this paper reports the results of a study that utilised the SOLO taxonomy in assessing students' mathematical problem-solving abilities. It also highlights how the SOLO taxonomy can influence instructional and assessment practices in fostering students' mathematical problem-solving on quadratic equations and related algebraic concepts.

## METHODS

### Participants

The participants were 80 Grade 11 students (52 females and 28 males) aged between 15 and 19 years (M = 16.6, SD = .851). All the 80 students came from one Grade 11 class at a public secondary school in Ndola district of Zambia. The participants were purposively selected because they belonged to the largest among all the Grade 11 classes at the participating school. Limitations in financial resources also made it difficult to involve other teachers and their classes. All participants provided written consent, and permission to collect data from the said participants was granted by relevant authorities.

### Materials

A mathematical problem-solving test was administered to students immediately after they had completed all the lessons on quadratic equations. Test items were developed in line with items designed by Collis et al. (1986) on assessing students' problem-solving abilities. Their items were constructed by combining the SOLO technique and the idea of super-items. Similarly, the test items for this research were designed for the purpose of assessing each student's problem-solving ability in line with the increasing complexity of the response structure reflected in the SOLO categories.

Before administration to the participants, all the test items were validated by 7 mathematics educators at different levels (i.e. 2 secondary school teachers 2 college lecturers and 3 university lecturers). These participants were selected purposively because of their vast research experience and knowledge of the Zambian mathematics curriculum for secondary schools. Participants were requested to rate each test item in terms of sufficiency, clarity, coherence and relevance and to provide comments on how each of the items could be improved to fit the purpose of the present research. A brief description for each of these four indicators is given below:

i. Sufficiency: The items are adequate to measure students' mathematical problem-solving abilities on quadratic equations and related algebraic concepts.

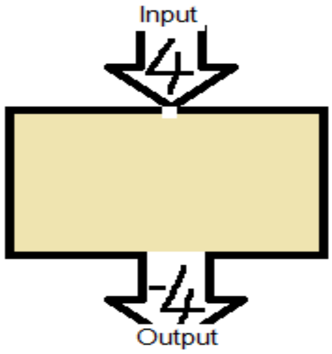
ii. Clarity: The item is well formulated and can be understood by Grade 11 students undertaking the Zambian secondary school mathematics course.

iii. Coherence: The item is logically related to the SOLO taxonomy level it is intended to measure

iv. Relevance: The item is very essential and important in assessing students' conceptual understanding of quadratic equations and related algebraic concepts.

After analysing all the comments from participants, the test items were revised and improved accordingly. Figure 1 shows an example of the items that were intended to assess students' mathematical problem-solving abilities using the SOLO taxonomy response structure. Allocation of students' responses for each problem-solving task was done in line with the five SOLO levels (pre-structural, unistructural, multi-structural, relational and extended abstract) as prescribed in the previous section of this paper. For instance, a student who fails to answer any of the problem-solving tasks shown in Figure 1 would be classified as operating at the pre-structural level. A student who manages to answer the problem-solving task (a) and fails the rest would be classified as operating at the unistructural level. Problem-solving task (b) corresponds to the multi-structural level while (c) corresponds to the relational level. A student who manages to answer all the problem-solving tasks from (a) to (e) would be classified as having attained the highest level (extended abstract) of the SOLO taxonomy.

The figure below represents a number processing machine. It squares the number you put in and then subtracts that same number five times. For instance, if you put in 4, it brings out $-4$ as illustrated below:




**(a) If the machine brings out $-4$, what number was put in?**
**(b) What number would be displayed as output if 5 is the input?**
**(c) Determine two possible inputs when the output from the machine is 6.**
**(d) Given that the number $y$ comes out of the machine when the number $x$ is put in, write down the equation connecting $x$ and $y$.**
**(e) Based on your equation in (d) above and assuming that $y = 36$, determine two possible values of $x$**

Figure 1: Questions depicting the SOLO taxonomy levels (Adapted from Collis, Romberg, & Jurdak, 1986)

**Procedure**

Data to address our main research question were collected in two phases. Firstly, a pre-test was administered and analysed. Remedial lessons were then prepared and presented with some modifications to earlier instructional and assessment approaches used. Both the researcher and the teacher had to sit down and design lessons with Student-Teams Achievement Division (STAD) model of cooperative learning taking the centre stage (see Slavin, 2015). Instead of the dominant *"hear lecture, do problems, get feedback"* order of affairs as earlier observed, remedial lessons were characterised by having students to learn in heterogeneous groups of six students. Each group comprised of two high performing, two moderate performing and two low performing students while taking gender issues into consideration. We hypothesised that by having students to learn mathematics in such an environment, the more knowledgeable students would assist their peers in understanding the topic at hand.

After the conduct of remedial lessons, a post-test was administered to the same group of students. Descriptive statistics (frequencies, mean, standard deviation and standard error) and inferential statistics (dependent samples t-test) were utilised in the data analysis process. We are very much aware of the shortcomings of the design (one-group pre- and post-test design) implemented in this study especially the lack of randomisation and other threats to internal validity (Fraenkel, Wallen, & Hyun, 2006). However, our main area of interest was to understand students' mathematical problem-solving abilities with special reference to the SOLO taxonomy levels. This enabled the researcher and the teacher to sit down, reflect on students' difficulties and misconceptions, and then modify instruction and assessment approaches with the aim of fostering mathematical problem-solving ability among learners.

## RESULTS

As earlier indicated, the primary purpose of this research was to examine how the SOLO taxonomy can influence assessment and instructional approaches in fostering students' mathematical problem-solving abilities. Table 1 and Figure 2 illustrate the frequency associated with each SOLO level before and after the intervention.

Table 1
*Respondents' frequency for five levels of the SOLO taxonomy*

| Level | Category | Pre-test | | Post-test | |
|---|---|---|---|---|---|
| | | **Frequency** | **Percent** | **Frequency** | **Percent** |
| 1 | Pre-structural | 8 | 10.0 | 3 | 3.8 |
| 2 | Unistructural | 49 | 61.3 | 12 | 15 |
| 3 | Multi-structural | 6 | 7.5 | 14 | 17.5 |
| 4 | Relational | 11 | 13.8 | 21 | 26.3 |
| 5 | Extended abstract | 6 | 7.5 | 30 | 37.5 |
| Total | | 80 | 100.0 | 80 | 100 |

Results displayed in Table 1 indicate that the majority of students (71.3%) were classified under the two lowest (pre-structural and unistructural) levels of the SOLO taxonomy during the pre-test. Only 28.7% of the participants' solutions were associated with multi-structural, relational and extended abstract levels of the SOLO taxonomy. This implies that the majority of students exhibited a limited understanding of quadratic equations and demonstrated inadequate mathematical problem-solving ability despite having learned the topic in full. Results displayed in Table 1 also indicate that 8 (or

10%) of the participants had their solutions rated at pre-structural level during the pre-test. This clearly suggests that those students either failed to interpret the information supplied in the stem of the problem situation or lacked the basic understanding of arithmetic and algebra.

After an appropriate intervention and remedial lessons, students' mathematical problem-solving abilities increased as most of them (63.8%) managed to reach relational and extended abstract levels of the SOLO taxonomy compared to the earlier 21.3%. The pictorial representation of these results in Figure 2 also shows that more students attained higher levels of SOLO taxonomy during the post-test than that of the pre-test. On the other hand, the number of students operating at lower levels of SOLO taxonomy reduced dramatically during the post-test compared to that of a pre-test (refer to Figure 2). For instance, the percentage of responses allocated to the unistructural level reduced from 61.3% during the pre-test to 15% during the post-test. This reflects an improvement in students' mathematical problem-solving abilities during the post-test.

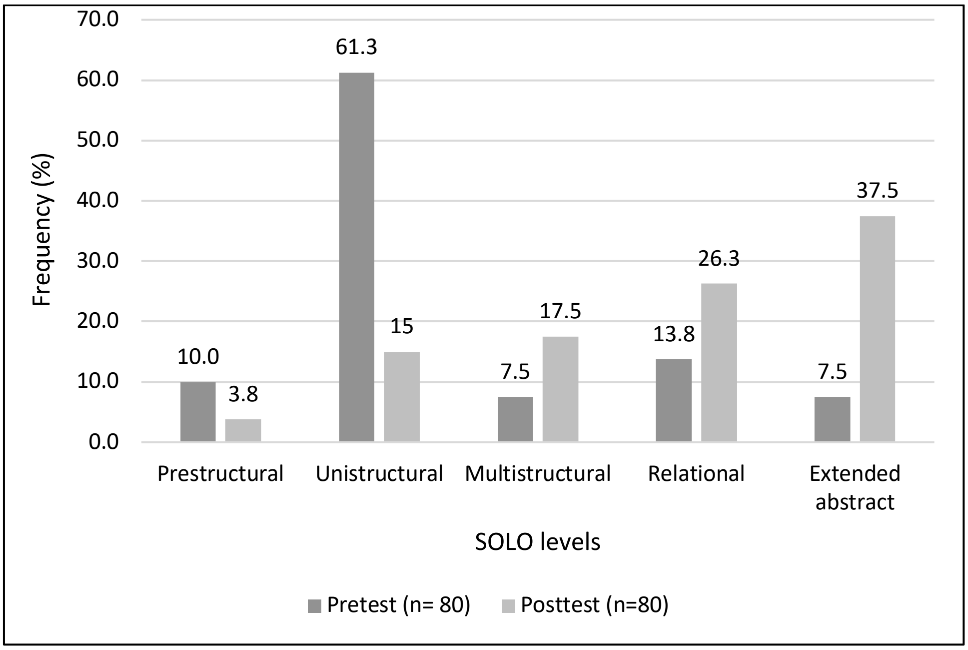



Figure 2: Respondents' frequency for five levels of the SOLO taxonomy

To test the statistical significance of the difference in scores between pre-test and post-test, a paired samples t-test was conducted. Before the conduct of this statistical test, assumptions were checked and none of them was violated except for that of randomisation as indicated in the methods section. Following recommendations by Field (2009), a Kolmogorov-Smirnov (K-S) normality test was performed on score differences of the paired values (pre-test and post-test) giving $D(80) = 0.086, p = 0.200$. Since the K-S normality test showed insignificant results (because $p > 0.05$), it was concluded that the distribution of the test scores were not significantly different from normal. This gave an assurance that a paired samples t-test was the right technique to perform in determining whether there was a statistical evidence that the mean difference between the pre-test and post-test was significantly different from zero. Table 2 and Table 3 illustrate results for descriptive statistics and the paired samples t-test respectively.

Table 2

*Descriptive statistics of the paired samples*

| **Measure** | **Mean** | **Sample size** | **Std. Deviation** | **Std. Error** |
|---|---|---|---|---|
| Pretest score | 32.00 | 80 | 17.782 | 1.988 |
| Posttest score | 53.75 | 80 | 20.951 | 2.342 |

Table 3

*Paired samples t-test*

| | **Paired Differences** | | | | | **t** | **df** | **Sig. (2-tailed)** |
|---|---|---|---|---|---|---|---|---|
| | | | | ***95% C.I*** | | | | |
| | ***M*** | ***SD*** | ***SE*** | ***Lower*** | ***Upper*** | | | |
| Pretest score - Posttest score | -21.75 | 19.86 | 2.22 | -26.17 | -17.33 | -9.796 | 79 | .000 |

On average, participants performed significantly better in the post-test $(M = 53.75, SE = 2.34)$ compared to pretest $(M = 32.0, SE = 1.99), t(79) = -9.796, p < .05, r = .74$. Results displayed in Table 3 further shows that the post-test scores were 21.75 points higher than the pre-test scores $(95\%\ CI\ [-26.17, -17.33])$. According to the benchmarks for effect sizes as guided by Field (2009) and Rosnow and Rosenthal (2005), this value $(r = .74)$ represents an adequate and large effect (especially that it is greater than the threshold of $.5$ for a larger effect). Based on these results, it was inferred that besides being statistically significant, this effect is large enough to represent a significant improvement in students' mathematical problem-solving after the intervention.

## DISCUSSION AND IMPLICATIONS OF RESEARCH FINDINGS

Based on these findings, it is highly recommended that teachers should make use of the SOLO taxonomy in assessing learners' mathematical problem-solving, reasoning and understanding. Results presented in this paper also echo previous findings (e.g. Jurdak & Mouhayar, 2013; Lim & Wun, 2012; Mulbar et al., 2017) in demonstrating the need for teachers to make use of the SOLO taxonomy in order to identify deficiencies in students' mathematical thinking, problem-solving and algebraic reasoning. This, in turn, could provide them with insights into appropriate measures to undertake in fostering students' understanding of mathematical concepts.

It has also been established that the majority (71.3%) of students were operating at the two lower levels of SOLO taxonomy before the intervention despite having been taught the topic in full. Further qualitative analysis of students' scripts falling in this category revealed that most of them did not understand the guidelines provided in the stem of the problem situation. By looking at the example given, this group of students made a wrong interpretation of simply negating the input to get the output and vice versa. Students' failure to engage with a given problem situation in an appropriate way has been reported by the Chief Examiner (Examinations Council of Zambia, 2012, 2015, 2018), as one of the contributing factors to candidates' failure to answer certain questions correctly. To mitigate such a challenge, Ojose (2008) has provided guidance worth heeding: teachers need to encourage their students to begin by identifying and analysing the major elements of a given task/question as doing so would enable them to extract the information needed in solving a particular task.

Another significant inference drawn from the results presented in this paper is that students at the same grade level, exposed to similar conditions may not possess the same cognitive ability in solving a particular problem-solving task. This partly confirms the remarks by Eggen and Kauchak (2000) on Piaget's four-stage theory that a student's cognitive ability to undertake mathematical tasks should not be tied to his/her age. By applying the SOLO taxonomy, teachers can understand the levels at which their students are operating concerning a particular mathematical concept. This is quite critical to teachers because it may help them to make informed decisions as they search for appropriate instructional and assessment practices for fostering students' mathematical problem-solving abilities.

Contrary to the perceptions of some teachers that cooperative learning cannot be successful in large classes, this study has revealed that the STAD model of cooperative learning can increase students' understanding of mathematical concepts even in a class of 80 students. This could be attributed to the fact that learning is decentralised to small cooperative groups instead of a teacher focusing on the entire class as a whole. In line with the social constructivist approach to teaching, Cooper (1995) also argued that cooperative learning enables students to take responsibility of their own learning and thus become actively involved in knowledge construction.

## CONCLUSION

The purpose of the case study presented here was to provide answers to the question of how the SOLO taxonomy can influence assessment and instructional approaches in fostering students' mathematical problem-solving abilities. Results have demonstrated that SOLO taxonomy is a necessary assessment tool in mathematics teaching and learning in the sense that: (i) it provides direction on the needy areas of focus in responding to students' difficulties with the subject matter, (ii) it gives an indication on the possibility of remedial work, (iii) it may trigger some pedagogical changes in order to switch to the ones that seem appropriate for addressing students' learning difficulties, and (iv) it provides teachers with an opportunity to reflect on areas that require improvement in their teaching and assessment practices. Therefore, there is a need for further research on how SOLO taxonomy can be used to assess and enhance students' understanding of different mathematical concepts at different levels of education and different settings.

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